Accurate Digital Phase Estimation Process for Coherent Detection Using a Parallel Digital Processor

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Abstract *A phase estimation method is presented and demonstrated experimentally which is suitable for coherent detection of signals using a parallel digital processor. The method achieves an estimate within 0.1dB of the MAP estimate.*

Introduction

Coherent detection has re-emerged recently as a promising way to receive optical communications signals. In the latest implementations phase diverse detection is used, and the detected signals are digitised and then processed as numerical values to obtain a representation of the electric field of the signal. The use of a digital signal processor (DSP) to perform the complex operations to recover the signal makes it possible for coherent detection to be cost-effective compared to direct detection for some applications. Recent experiments using sampled coherent detection in a burst mode have demonstrated an information density of 2.5b/s/Hz [1], and a receiver sensitivity just 2.7dB higher than the theoretical minimum [2]. Also it has been shown that chromatic dispersion can be compensated in the digital domain with zero net penalty [2].

It is expected that a real-time DSP must process the digital signal values in parallel, given that the A/D converter sampling rate is higher than the clock frequency of the DSP core. A key issue in implementing sampled coherent detection is the estimation of the signal phase within the digital processor. The usual way to estimate the phase is with a phase locked loop (PLL), and it is well known that the PLL is intolerant to delay in the feedback loop [3]. The operation of demultiplexing the incoming signal digital values into parallel streams is equivalent to introducing a substantial delay, which precludes the PLL architecture. Noé has proposed an alternative phase estimation algorithm which employs no feedback [4], and it has been tried by Tsukamoto et al. [1], but their experiment used relatively narrow linewidth lasers for the signal and local oscillator (LO). The all-feedforward method uses planar filtering of a rectangular window of phasor samples, which is a convenient and not an optimum window shape. Differential logical decoding is used to avoid accumulating errors from cycle slips. In this paper a phase estimation method will be presented that uses feedback but can be implemented in a processor with any degree of parallelism. The averaging window is the optimum shape extending to infinity back in time.

Phase estimation algorithm

The DSP deals with quantities which represent samples taken at the symbol centres. In this paper such quantities will be written as $x(n)$, referring to the continuous-time quantity at time $n\tau_s$, where τ_s is the symbol period and $n = 0,1,2,\cdots$. A Lorentzian-shaped laser peak contains (unwrapped) phase noise $\phi(n)$ such that

$$\phi(n) = \phi(n-1) + w(n)$$

where $w(n)$ is a zero-mean Gaussian noise sequence having variance $\sigma_w^2 = 2\pi\tau_s\Delta\nu$, and $\Delta\nu$ is the linewidth. The received signal comprises the data-modulated signal plus additive noise, and can be written in normalised form as

$$r(n) = d(n)\exp(i\phi(n)) + p(n)$$

$d(n)$ is the data; when M-ary PSK is used it takes on values from the set $\sqrt[M]{1}$; $p(n)$ is the complex additive Gaussian noise where each part has variance σ_p^2. To remove the data modulation $r(n)$ is raised to Mth power

$$s(n) = r(n)^M = \exp(iM\phi(n)) + Mp'(n)\exp(iM\phi(n)) + O(p^2)$$

where $p'(n)$ is a different Gaussian noise with variance σ_p^2. Given that the noise is small, a small angle expansion applies

$$\theta(n) = \arg(s(n)) \approx M(\phi(n) + \text{Im}[p'(n)]) \qquad (1)$$

Stated this way, the problem of finding $\hat{\phi}(n)$, the best estimate of $\phi(n)$, is a Wiener filtering problem.

$$\hat{\phi}(n) = \frac{1}{M}\hat{\theta}(n)$$

The zero lag Wiener filter is calculated using the procedures of reference 5, and in terms of its z-transform the filter is

$$\hat{\theta}(z) = \frac{1-\alpha}{1-\alpha z^{-1}}\theta(z) \qquad (2)$$

where

$$\alpha = \frac{\sigma_w^2 + 2\sigma_p^2 - \sigma_w\sqrt{\sigma_w^2 + 4\sigma_p^2}}{2\sigma_p^2}$$

The Wiener filter having lag of D symbols is

$$\hat{\theta}(z) = \frac{1-\alpha}{1-\alpha z^{-1}}\left(\alpha^D + (1-\alpha)\sum_{k=1}^{D}\alpha^{D-k}z^{-k}\right)\theta(z) \qquad (3)$$

The finite lag filter gives a better result than the zero lag filter because it considers D symbols into the future as well as the infinite past in making its estimate. In equation 1 $\theta(n)$ is implicitly an unwrapped phase, but $\arg(s)$ gives only wrapped values in the range $-\pi$ to π. Phase unwrapping can be implemented by modifying equation 1

$$\theta(n) = \arg(s(n)) + g(n) \qquad (4a)$$

$$g(n) = g(n-1) + 2\pi\, f(\arg(s(n)) - \arg(s(n-1))) \quad (4b)$$

where $\quad f(x) = 1$ if $x > \pi$

$$f(x) = 0 \ \text{ if } \ |x| < \pi$$

$$f(x) = -1 \ \text{ if } \ x < -\pi$$

$g(n)$ keeps count of the phase cycles, and needs only to be recorded modulo M.

The application of equations 2 and 4, or 3 and 4, is expected to give a very accurate phase estimate since the Wiener filter is the least mean square error linear estimate, and the approximations applied have been minor. However the equations cannot be implemented directly in the DSP because they involve feedback of the immediately preceding result, which is not available in the parallel architecture. Equations 2 and 3 can be recast using a look-ahead computation [6] so as to refer to feedback of a distant past result, L symbols in the past, by multiplying numerator and denominator by a polynomial. The zero lag estimate equation 1 becomes

$$\hat{\theta}(z) = \frac{1-\alpha}{1-\alpha^L z^{-L}} \sum_{k=0}^{L-1} \alpha^k z^{-k}\ \theta(z)$$

The look-ahead version of the cycle count equation 3b is

$$g(n) = g(n-L) + 2\pi \sum_{k=0}^{L-1} f(\arg(s(n-k)) - \arg(s(n-k-1)))$$

The finite lag equations can be recast in a similar way, but the result is not listed here because of limited space.

Experiment & results
The phase estimation algorithms were tested by using a long burst of data (4.1×10^6 samples) of a noisy BPSK signal ($M=2$) where the signal and LO were DFB lasers having 48MHz combined linewidth. The symbol rate was 1.5GBaud, deliberately low to make $\tau_s \Delta\nu$ a high value of 0.032. The experimental layout is shown in figure 1, and the method is the same as used in reference 2.

Figure 2 shows the Q-factor obtained by applying different phase estimation algorithms to the same measurement burst. The Q-factor is derived from the bit error rate after a differential operation. The results for the Wiener phase estimates were each calculated both using the formulas with immediate feedback ($L=1$) and for $L=32$, and found to give identical

results. The maximum a posteriori (MAP) phase estimate is the best possible estimate, and was calculated by trying for each symbol all possible data sequences for the next 5 symbols, calculating the phase by Newton's approximation, and selecting the most probable option. The $D=10$ Wiener estimate is only 0.1dB worse than the MAP estimate. The digital PLL estimate is 2.7dB behind that, and of course it cannot be implemented in a parallel DSP.
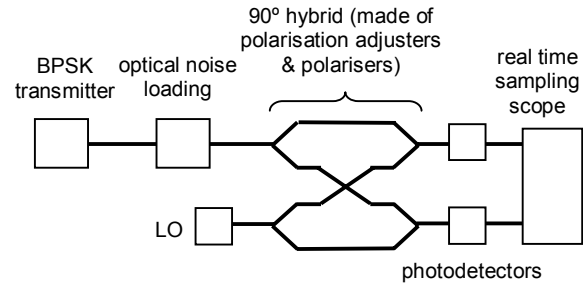

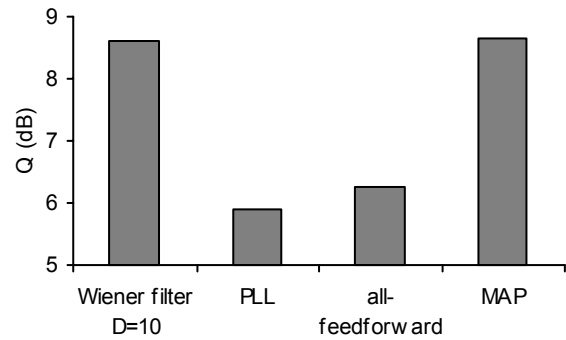**Figure 1** *Apparatus to detect BPSK signal by sampled coherent detection.*
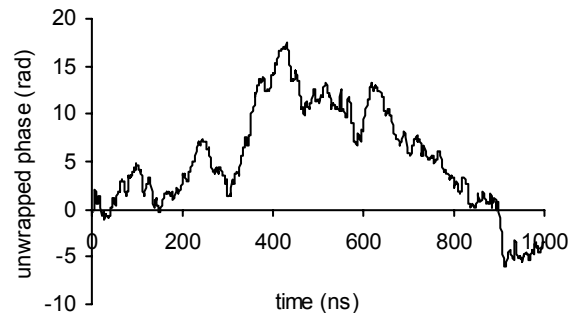

Figure 2 *Comparison of phase estimation methods.*

Figure 3 *Example of phase of Wiener filter case.*

References
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